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Analytical Modeling of Nonradial Expansion Plumes

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Introduction

THE Simons model[1,2] is a simple analytical model used in the calculation of axisymmetric rocket and thruster exhaust plume flowfields. The model proposes that the density ρ at the point (r,θ) in the axisymmetric plane is given by

$$\rho(r,\theta) = \frac{A}{r^2} f(\theta) \tag{1}$$

where A is the plume constant and $f(\theta)$ is the angular density distribution. It is therefore assumed that the density diverges radially. A description of the procedures for the determination of A and $f(\theta)$ for a particular nozzle geometry and gas type are included in Ref. 2.

Due to its analytical nature, the Simons model is a widely used engineering tool. It is often employed, for example, in the swift calculation of impingement effects associated with a satellite attitude control thruster system.[3]

It has been shown by Boyd and Stark[4] that, for certain plumes of nitrogen expanding from small nozzles, the assumption of radial decay gives rise to unsatisfactory results in calculations made with the Simons model. Errors of up to 50% were noted for the prediction of the density distribution along the plume axis for which experimental data was available. This phenomenon was observed to occur for nozzles having a large exit Mach number and/or a large nozzle exit half-angle. By making successful method of characteristics calculations of similar plumes, a new analytical formulation was derived. This new method has been termed the Modified Simons model and allows the nonradial nature of such plumes to be successfully predicted. Specifically, the assumption of radial expansion is replaced in the Modified Simons model by the following:

$$\rho(r,\theta) = \frac{A}{r^2 - ar + b} f(\theta) \tag{2}$$

where

$$a = 3\theta_E^{1/2} Ma_E r_E \tag{3a}$$

$$b = 5\theta_E Ma_E^2 r_E^2 \tag{3b}$$

In Eqs. (3), θ_E is the nozzle exit half-angle expressed in radians, Ma_E the nozzle exit Mach number, and r_E the nozzle exit radius. The exit Mach number is obtained by iterative solution of the following expression for the ratio of the area at the nozzle exit to that at the nozzle throat:

$$\epsilon = \frac{1}{Ma_E} \sqrt{\left[\frac{1 + \frac{\gamma-1}{2} Ma_E^2}{1 + \frac{\gamma-1}{2}}\right]^{\frac{\gamma+1}{\gamma-1}}} \tag{4}$$

where γ is the ratio of specific heats. The area at the nozzle exit is that reduced by the presence of any boundary-layer thickness along the nozzle wall. Under certain conditions it is possible for the denominator in Eq. (2) to become negative. Generally, the behavior of the model at distances of less than 5 exit radii from the nozzle is not reliable.

While these expressions have been derived for the expansion of nitrogen, which has $\gamma = 1.4$, it was shown in Ref. 4 that their application to a hydrazine plume, in which $\gamma = 1.37$, also gave significant improvement on the original Simons model when compared with experimental data. The expressions have therefore been found to be of great value in improving the prediction of plume flows for gases in which the ratio of specific heats is close to that for a diatomic gas.

For most expansion plume investigations undertaken in the laboratory, the test gas is nitrogen. But in real propulsion applications the use of solid fuel and bipropellant systems is more common. Depending on the specific fuels employed, the ratio of specific heats may be very different from 1.4. It is therefore the purpose of the present study to determine the usefulness of the Modified Simons model as applied to monatomic and polyatomic gas expansions.

Calculations

The expansion of nitrogen (N_2), argon (Ar), and tetrafluoromethane (CF_4) through a number of nozzles has been investigated experimentally by Legge et al.[5] One of the nozzles considered is the MBB/ERNO 0.5N thruster. This engine is designed to burn monopropellant hydrazine and its expansion plume has received detailed analysis by Boyd and Stark.[6] In the present study, calculation is made of the expansion of the three test gases through this nozzle and the results are compared with experimental data obtained along the plume axis.[7]

Although the nozzle geometry is fixed, the structure of the exhaust plumes for the three different gases is quite different. In addition to having different values for γ (see Table 1), viscosity effects along the nozzle wall will also be important. For the small dimensions of this thruster (the exit radius is less than 3 mm), a thick laminar boundary layer exists at the nozzle exit plane. The reduction in volume of the inviscid flow at the nozzle exit plane reduces the exit Mach number as calculated by Eq. (4), and thus has a direct effect on Eqs. (3). It is this aspect of such calculations that makes difficult the derivation of a more generalized Modified Simons model. The calculated exit plane quantities, together with the plume constant employed, are listed in Table 1 for each of the gases.

In Fig. 1 the distribution of Pitot pressure along the plume axis is shown for the expansion of nitrogen. The improvement provided by the Modified Simons model is quite apparent. Calculations made with the original Simons model are found to be in error by as much as 35% when compared with the experimental data obtained in the near field of the plume. Although the two sets of theoretical solutions are observed to converge at larger distances along the axis, there is still an error of 11% associated with the original Simons model result obtained at the last available experimental data point. The corresponding value obtained with the Simons model shows a deviation of just 3% from the experiment.

The expansion of monatomic argon is now considered. The ratio of the theoretical predictions of pitot pressure to the values obtained experimentally are plotted in Fig. 2. The improvement attained with the Modified model in comparison to the results obtained with the original Simons model are quite satisfying. A significant improvement in the results is found particularly at larger distances from the nozzle exit plane. Similar plots for tetrafluoromethane are shown in Fig. 3. Once

Received Oct. 24, 1988; revision received March 13, 1989.

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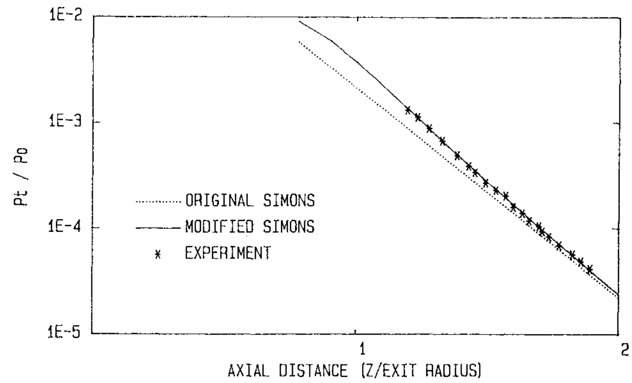

Fig. 1 Pitot pressure distributions along the plume axis for the expansion of nitrogen through the MBB/ERNO 0.5N thruster.

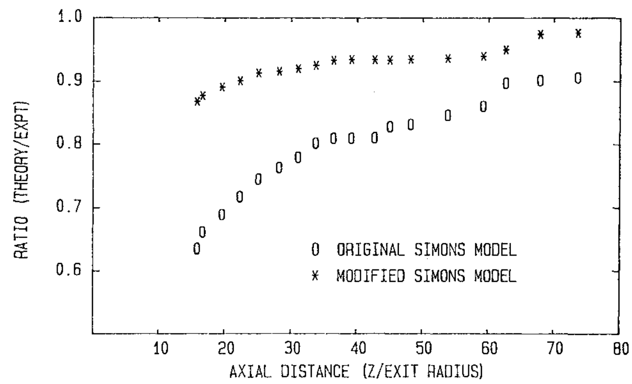

Fig. 2 Ratio of theoretical predictions to experimental data for pitot pressure distribution along the axis for the expansion of argon.

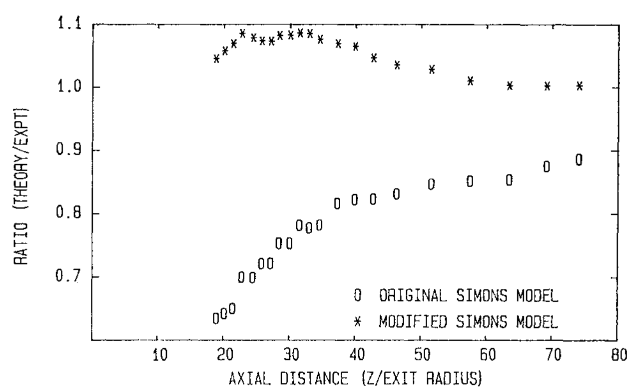

Fig. 3 Ratio of theoretical predictions to experimental data for pitot pressure distribution along the axis for the expansion of tetrafluoromethane.

again it is clear that the expressions derived for the expansion of nitrogen are usefully applied to the polyatomic gas.

It is interesting to compare Figs. 2 and 3. In the case of the monatomic gas, the use of Eqs. (3) tends to just overestimate the pitot pressure in comparison with experiment. On the other hand, when γ is much less than 1.4, the results are underestimated by the theoretical predictions. If such behavior is confirmed by further comparison with experimental data, then it may be possible to derive an improved form of Eqs. (3) as a function of the specific heat of the gas.

In Ref. 4 it was possible to define the nozzle exit conditions under which the use of the modified Simons model is preferred to the original formulation. The criterion employed was a divergence in axial density of 10% for the two methods, and was expressed as a function of exit Mach number and nozzle exit half-angle. For the expansion of monatomic and polyatomic gases, and also for gas mixtures, it is suggested that the decision be made with reference to an exit Mach number obtained with $\gamma = 1.4$ together with the exit half-angle and the usage criterion chart shown in Ref. 4.

Table 1 Nozzle conditions investigated

Test gas	N_2	Ar	CF_4
Molecular weight	28	40	88
Ratio of specific heats	1.40	1.67	1.17
Boundary-layer thickness, %	34	35	27
Mach number	5.09	7.23	3.96
Plume constant	3.39	6.78	0.93

It is perhaps surprising that Eqs. (2) and (3), which were derived from a diatomic gas, should be so effective when applied to monatomic and polyatomic molecules. The physical explanation for the nonradial behavior lies in the fact that the nozzle exit conditions are markedly different from those for an effusive source. It is shown in Ref. 4 that the form of Eqs. (3) is at least quantitatively correct.

Conclusions

The nonradial density decay observed experimentally in jets of nitrogen, argon, and tetrafluoromethane has been successfully predicted using the Modified Simons model. Calculations made with this simple analytical model offer significant improvement on the results obtained with Simons original formula. It is therefore proposed that the Modified Simons model may be applied with some confidence to any rocket or thruster plume that may be expected to exhibit a significant degree of nonradial behavior.

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Further Investigations of Fracture Toughness Measurement Using Numerical-Experimental Technique

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Introduction

THE theory of elastic-plastic fracture mechanics has been widely employed for efficient and reliable design of struc-

Received Jan. 9, 1989; revision received March 15, 1989.

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